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COSET CONSTRUCTION OF SUPERSTRINGS VIA THE COADJOINT ORBIT METHOD

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The previously proposed general construction of geometric actions on infinite-dimensional group coadjoint orbits in terms of fundamental group one-cocycles is applied to provide an alternative formulation of the Green-Schwarz superstring. It is shown that the latter model can be consistently constructed as geometric action on a certain infinite-dimensional coset space of the semi-direct product of left- and right-handed Virasoro groups with a Kac-Moody group based on an appropriate modification of the super-Heisenberg-Weyl group.

1. Introduction

Covariant quantization of superstrings with manifest space-time supersymmetry (Green-Schwarz (GS) superstrings[1]) has remained, since their discovery, a major challenging problem in modern field theory. Two different approaches were proposed so far to deal with the principal difficulty consisting in an incompatibility between manifest super-Poincaré invariance and the famous fermionic κ-gauge symmetry.[2,1] The first approach (the "harmonic superspace" formalism; see Ref. 3 and references therein) solves the problem by introducing a finite number of auxiliary variables and requires a finite number of ghosts in the covariant BFV-BRST[4,5] formalism. The second approach[6] employs an infinite tower of ghosts-for-ghosts and requires certain (*ad hoc*) regularization of divergent sums over these ghosts. Both approaches, however, have not yet provided a sufficiently tractable formalism for covariant superstring amplitude calculations.

Thus, it seems reasonable to look for alternative approaches trying to resolve the difficulties inherent in the BFV-BRST covariant quantization scheme for the GS superstring. Recently, it was suggested in Ref. 7 to reformulate the GS superstring model within the group coadjoint orbit method.[a] This idea is quite natural, since, as

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[a] Some basic references for the recent revival of the coadjoint orbit method[8] in the context of its extension to infinite-dimensional groups and applications to $D = 2$ conformal field theories are in Refs. 9, 7, 10, and 11.

any other $D = 2$ conformal field theory, the GS superstring model possesses an infinite-dimensional group of Noether symmetries, which one expects to completely determine the whole dynamics. Still, the complete coadjoint orbit reformulation of the GS superstring requires care in choosing the correct underlying infinite-dimensional algebra, which must *close*. It is the resolution of this problem which is the main concern of the present letter.

In what follows, we first briefly recapitulate the basic formulae of a recently proposed[12–15] unified treatment of geometric actions on infinite-dimensional group G co-orbits. In particular, we describe within the above formalism the fundamental group composition laws generalizing the famous Polyakov-Wiegmann composition laws for the WZNW models.[17] Subsequently, we briefly discuss the generalization to the case of geometric actions on co-orbits of semidirect products $G_1 \ltimes G_2$, as well as geometric actions on infinite-dimensional (left) cosets $H\backslash G$ (i.e., both G and the subgroup H being infinite-dimensional). Further, we consider the geometric action on a generic orbit of the infinite-dimensional group:

$$G = (\mathrm{Vir})^2 \ltimes (\mathcal{MSHW})\,. \tag{1}$$

The first factor $(\mathrm{Vir})^2$ in the semidirect product (1), denotes the direct product of two Virasoro groups (left- and right-handed), while the second factor $(\mathcal{MSHW})$ denotes the Kac-Moody group based on an appropriate modification of the super-Heisenberg-Weyl group. We explicitly present the fundamental group one-cocycles, entering the general construction of the geometric actions, in the case of (1) and show how to obtain the standard GS superstring action as geometric action on a special infinite-dimensional coset space of (1).[b]

2. Geometric Actions on Group Coadjoint Orbits and Coset Spaces

Let G be an infinite-dimensional Lie group, $\mathcal{G}$ – its Lie algebra and $\mathcal{G}^*$ – the dual vector space of $\mathcal{G}$ with respect to the natural bilinear form $\langle B \,|\, \xi \rangle$, $\xi \in \mathcal{G}$, $B \in \mathcal{G}^*$. Let $\tilde{\mathcal{G}} = \mathcal{G} + \mathbb{R}$ be a central extension of $\mathcal{G}$ and $\tilde{\mathcal{G}}^*$ be the dual space of $\tilde{\mathcal{G}}$. The adjoint actions of G and $\mathcal{G}$ on $\mathcal{G}$, given by $\mathrm{Ad}(g)\xi = g\xi g^{-1}$ and $\mathrm{ad}(\xi)\eta = [\xi, \eta]$, and the corresponding coadjoint actions on $\mathcal{G}^*$, given by $\langle \mathrm{Ad}^*(g)B \,|\, \xi\rangle = \langle B \,|\, \mathrm{Ad}(g^{-1})\xi\rangle$ and $\langle \mathrm{ad}^*(\xi)B \,|\, \eta\rangle = -\langle B \,|\, [\xi, \eta]\rangle$, can be extended to $\tilde{\mathcal{G}}$ and $\tilde{\mathcal{G}}^*$ respectively as follows,

$$\tilde{\mathrm{A}}\mathrm{d}(g)\,(\xi, n) = (\mathrm{Ad}(g)\xi,\; n + \lambda\langle S(g^{-1}) \,|\, \xi\rangle)\,, \tag{2}$$

$$\tilde{\mathrm{a}}\mathrm{d}_{(\xi_1, n_1)}(\xi_2, n_2) \equiv [(\xi_1, n_1), (\xi_2, n_2)] = ([\xi_1, \xi_2],\, -\lambda\langle s(\xi_1) \,|\, \xi_2\rangle)\,, \tag{3}$$

$$\tilde{\mathrm{A}}\mathrm{d}^*(g)\,(B, c) = (\mathrm{Ad}^*(g)B + c\lambda S(g),\, c)\,, \tag{4}$$

$$\tilde{\mathrm{a}}\mathrm{d}^*(\xi,\, n)\,(B, c) = (\mathrm{ad}^*(\xi)B + c\lambda s(\xi),\, 0)\,. \tag{5}$$

[b] A reformulation of the GS superstring as an asymmetric WZNW sigma-model on finite-dimensional coset was proposed in Ref. 16.

In Eqs. (2)–(5) $(\xi, n) \in \tilde{\mathcal{G}}$, $(B, c) \in \tilde{\mathcal{G}}^*$ and λ denotes a normalization constant specific for the model.

The group property $\tilde{\mathrm{A}}\mathrm{d}(g_1 g_2) = \tilde{\mathrm{A}}\mathrm{d}(g_1)\,\tilde{\mathrm{A}}\mathrm{d}(g_2)$ of (2) implies the following property of $S(g)$,

$$S(g_1 g_2) = S(g_1) + \mathrm{Ad}^*(g_1)\, S(g_2)\,. \tag{6}$$

Equation (6) is the mathematical expression of the statement that $S(g)$ is a $\mathcal{G}^*$-valued one-cocycle on G. From (2) it is clear that $S(g)$ is precisely the "integrated anomaly", i.e., the "anomaly" for finite group transformations $g \in G$ due to the central extension part in the Lie algebra $\tilde{\mathcal{G}}$. In Eqs. (3) and (5) $s(\cdot)$ denotes the infinitesimal part of $S(\cdot)$: $S(I + \xi) = s(\xi) + O(\xi^2)$, $\xi \in \mathcal{G}$, i.e., $s(\cdot)$ represents the usual Lie-algebra $\mathcal{G}$-"anomaly".

There is another fundamental object – a $\mathcal{G}$-valued Maurer-Cartan one-form $y(g)$ on the group G:

$$dy(g) = \frac{1}{2}\,[\,y(g),\ y(g)] \tag{7}$$

(d indicating exterior derivative) which satisfies a similar cocycle condition as (6):

$$y(g_1 g_2) = y(g_1) + \mathrm{Ad}(g_1)\, y(g_2)\,. \tag{8}$$

$S(g)$ and $y(g)$ are related (each one being a nonlinear non-local functional of the other) through the following basic equation,

$$dS(g) = \mathrm{ad}^*(y(g))\, S(g) + s(y(g))\,. \tag{9}$$

A coadjoint orbit of G is a submanifold of the vector space $\tilde{\mathcal{G}}^*$ of the form $\mathcal{O}_{(B_0,c)} \equiv \{\tilde{\mathrm{A}}\mathrm{d}^*(g)\,(B_0, c);\ \forall g \in G\}$, where (B_0, c) is a generic point in $\tilde{\mathcal{G}}^*$. It turns out that physically interesting models are described in terms of a special class of G co-orbits with $B_0 = 0$, i.e.,

$$\mathcal{O}_{(0,c)} \equiv \{\tilde{\mathrm{A}}\mathrm{d}^*(g)\,(0, c) = (c\lambda S(g), c);\ \ \forall g \in G\}\,. \tag{10}$$

The stationary subgroup G_{stat} and subalgebra $\mathcal{G}_{\mathrm{stat}}$ of the orbit $\mathcal{O}_{(0,c)}$, (10) are respectively

$$G_{\mathrm{stat}} = \{g_0 \in G;\ S(g_0) = 0\}\,, \quad \mathcal{G}_{\mathrm{stat}} = \{\eta_0 \in \mathcal{G};\ s(\eta_0) = 0\}\,, \tag{11}$$

and, according to the cocycle property (6), the orbit $\mathcal{O}_{(0,c)}$ is a homogeneous space of G with respect to the right group multiplication:

$$\mathcal{O}_{(0,c)} = G/G_{\mathrm{stat}}\,. \tag{12}$$

According to the general theory,[8] each G co-orbit is endowed with the natural Kirillov-Kostant-Souriau (KKS) symplectic structure and, therefore, it can serve as a phase space of a classical dynamical system. In the case of (infinite-dimensional) groups with central extension $\tilde{G}$ one can write the corresponding geometric action on $\mathcal{O}_{(0,c)}$ entirely in terms of the fundamental group one-cocycles $S(g)$ and $y(g)$ (related by the *off-shell* equation (9))[12–15]:

$$W[g] = -c\lambda \int \left[\langle S(g) | y(g) \rangle - \frac{1}{2} d^{-1} (\langle s(y(g)) | y(g) \rangle) \right]$$
$$= -\frac{1}{2} c\lambda \int d^{-1} (\langle s(y(g^{-1})) | y(g^{-1}) \rangle). \qquad (13)$$

In (13) the integral is over an one-dimensional curve on the phase space $O_{(0,c)}$ with parameter t. Accordingly, the exterior derivative along the curve becomes $d = dt\partial_t$, and the one form $y(g) = dt\, y_t(g)$. Also, since the phase space $O_{(0,c)}$ (12) is parametrized by all group coordinates $g \in G$, the stationary subgroup G_{stat} (11) plays the role of a gauge symmetry group.

The group cocycle relations (6) and (8) imply the following fundamental group composition laws[14,15] generalizing the Polyakov-Wiegmann composition laws[17] for the WZNW models to arbitrary G co-orbit models,

$$W[g_1 g_2] = W[g_1] + W[g_2] + c\lambda \int \langle S(g_2) | y(g_1^{-1}) \rangle. \qquad (14)$$

Specific examples of (14) for $D = 2$ (super)gravity are given in Refs. 18 and 15.

Among the numerous important applications of (14), they allow us to write down immediately the generalization of the geometric action (13) on G co-orbits to the case of co-orbits of semidirect products $G_1 \ltimes G_2$ and on (left) coset space $H \backslash G$ (let us stress that all groups involved are infinite-dimensional).

The general form of the geometric action on $H \backslash G$ reads[15]

$$W_{H\backslash G} \equiv W[h(g)^{-1} g] = W[g] - W[h(g)] + c\lambda \int \langle S(g) - S(h(g)) | y(h(g)) \rangle . \qquad (15)$$

One easily verifies that the coset action (15) is manifestly invariant under left group translation $g \to h_0 g$ for any $h_0 \in H$. In the particular case of "anomaly-free" subgroup H, i.e.,

$$\langle s(\xi_H) | \eta_H \rangle = 0 \text{ for any } \xi_H, \eta_H \in \mathcal{H}, \qquad (16)$$

the coset action (15) simplifies to

$$W_{H\backslash G} = W[g] + c\lambda \int \langle S(g) | y(h(g)) \rangle. \qquad (17)$$

Similarly, for a semidirect product $G = G_1 \ltimes G_2$ using the Mackey decomposition (see, e.g. Ref. 19)[c]:

$$g = g_1(g) g_2(g), \quad g_1(g) \in G_1, \quad g_2 \in G_2, \qquad (18)$$

one can explicitly express via the general composition laws (14) the G co-orbit action $W_G[g]$ in terms of the co-orbit actions $W_{G_{1,2}}[g]$ of the factors G_1 and G_2:

$$W_G[g] = W_{G_1}[g_1(g)] + W_{G_2}[g_2(g)] + c\lambda \int \langle S_G(g_2(g)) | y_{G_1}(g_1^{-1}(g)) \rangle. \qquad (19)$$

The action (19) may be regarded as coupling of "matter fields" (described by G_2) to "gauge" fields (described by G_1) via the last Noether-like local coupling term on

[c] On the level of Lie-algebras we have $\tilde{\mathcal{G}} = \mathcal{G}_1 + \mathcal{G}_2 + \mathbb{R}, [\mathcal{G}_1, \mathcal{G}_1] \subset \mathcal{G}_1 + \mathbb{R}, [\mathcal{G}_2, \mathcal{G}_2] \subset \mathcal{G}_2 + \mathbb{R}, [\mathcal{G}_1, \mathcal{G}_2] \subset \mathcal{G}_2$, i.e., $\mathcal{G}_2$ carries a representation of G_1.

the right-hand side of (19). Indeed, the infinitesimal form of the cocycle condition (8) $\delta_\eta y_{G_1}(g_1^{-1}) = -d\eta + [y_{G_1}(g_1^{-1}), \eta]$ (with $\eta \in \mathcal{G}_1$) implies that $y_{G_1}(g^{-1})$ is a "gauge potential" of the (infinite-dimensional) G_1, whereas the "integrated anomaly" $S(g)$ is a conserved current[13-15]: $dS(g)\,|_{\text{on-shell}} = 0$.

3. Superparticle

Before proceeding with the construction of the GS superstring via the coadjoint orbit method, let us consider as an warm-up exercise the coadjoint orbit construction of its zero-mode – the Brink-Schwarz superparticle.[20]

We start with the semidirect sum of the world-line reparametrization algebra and the D-dimensional super-Heisenberg-Weyl (SHW) algebra, which is spanned by the set of generators $\{\hat{T}, \hat{X}^\mu, \hat{P}^\mu, \hat{Q}^\alpha, \hat{\Theta}_\alpha\}$ with the following non-zero commutators,[d]

$$[\hat{X}^\mu, \hat{P}^\nu] = \eta^{\mu\nu}, \quad \{\hat{Q}^\alpha, \hat{Q}^\beta\} = 2i(\gamma^\mu)^{\alpha\beta}\hat{P}_\mu, \quad [\hat{Q}^\alpha, \hat{X}^\mu] = (\gamma^\mu)^{\alpha\beta}\hat{\Theta}_\beta\,,$$
$$\{\hat{Q}^\alpha, \hat{\Theta}_\beta\} = -i\delta^\alpha_\beta\,, \quad [\hat{T}, \hat{X}^\mu] = -\hat{P}^\mu\,. \tag{20}$$

The general form of a group element $g \in G(sp) = \{\hat{T}\} \ltimes (SHW)$ reads (with ξ – an element of the corresponding Lie-algebra)

$$g = \exp\xi = \exp\{\omega\hat{T} + p_\mu\hat{X}^\mu + q_\mu\hat{P}^\mu + \theta_\alpha\hat{Q}^\alpha + \chi^\alpha\hat{\Theta}_\alpha\}. \tag{21}$$

For the global group "anomaly" $S(g)$ and its infinitesimal form $s(\xi)$ we easily find

$$S(g) = \frac{1}{2}p^2\hat{T}^* + \left(q^\mu - \frac{1}{2}\omega p^\mu\right)\hat{X}^*_\mu - p^\mu\hat{P}^*_\mu + i\left(\chi^\alpha + \frac{3}{2}\not{p}^{\alpha\beta}\theta_\beta\right)\hat{Q}^*_\alpha + i\theta_\alpha\hat{\Theta}^{*\alpha}\,, \tag{22}$$

$$s(\xi) = q^\mu\hat{X}^*_\mu - p^\mu\hat{P}^*_\mu + i\chi^\alpha\hat{Q}^*_\alpha + i\theta_\alpha\hat{\Theta}^{*\alpha}\,. \tag{23}$$

In (22) and (23), $\hat{T}^*, \ldots, \hat{\Theta}^*$ denote the dual basis with respect to the basis $\hat{T}, \ldots, \hat{\Theta}$. According to (11), Eq. (22) yields the following stationary subgroup of the orbit $\mathcal{O}(sp)$ (10) of $G(sp)$,

$$G_{\text{stat}}(sp) = \{\hat{T}\}\,, \quad \text{i.e.,} \quad \mathcal{O}(sp) = \frac{\{\hat{T}\} \ltimes (SHW)}{\{\hat{T}\}}\,, \tag{24}$$

where $\{\hat{T}\}$ denotes the subgroup generated by $\hat{T}$.

The fundamental Maurer-Cartan form $y(g^{-1})$ has the following expansion,

$$-y(g^{-1}) = g^{-1}dg = \tilde{\omega}\hat{T} + \tilde{p}_\mu\hat{X}^\mu + \tilde{q}_\mu\hat{P}^\mu + \tilde{\theta}_\alpha\hat{Q}^\alpha + \tilde{\chi}^\alpha\hat{\Theta}_\alpha\,, \tag{25}$$

where the coefficients are one-forms. For the co-orbit action we get (with $c\lambda = 1$)

$$W_{\mathcal{O}(sp)}[g] = -\frac{1}{2}\int d^{-1}(\langle s(y(g^{-1}))|\,y(g^{-1})\rangle) = \int d^{-1}(\tilde{p}^\mu \wedge \tilde{q}_\mu + i\tilde{\theta}_\alpha \wedge \tilde{\chi}^\alpha). \tag{26}$$

[d] The indices μ, ν and α, β indicate Lorentz-vector and Majorana (Majorana-Weyl)-spinor indices in D-dimensional Minkowski space; $(\gamma^\mu)^{\alpha\beta} = (\gamma^\mu)^\beta_{\dot{\alpha}}\, C^{\alpha\dot{\alpha}}$ are the symmetric Dirac matrices.

The simplest way to find the explicit expressions for the coefficient one-form $\tilde{\omega},\ldots,\tilde{\chi}$ in (25) in terms of the group parameters of g (21) is to use the Maurer-Cartan equation (7). The result of a straightforward calculation reads

$$-y(g^{-1}) = d\omega\hat{T} + dp_\mu \hat{X}^\mu + (dq_\mu + i\theta\gamma_\mu d\theta - p_\mu d\omega) \times \hat{P}^\mu + d\theta_\alpha \hat{Q}^\alpha + (d\chi^\alpha + \not{p}^{\alpha\beta} d\theta_\beta)\hat{\Theta}_\alpha . \tag{27}$$

Substituting (27) into (26) we obtain

$$W_{O(sp)}[g] = \int dt \left[p^\mu (\dot{q}_\mu + i\theta\gamma_\mu \dot{\theta}) - \frac{1}{2}\dot{\omega}p^2 + i\chi\dot{\theta} \right]. \tag{28}$$

This action differs substantially from the usual superparticle action through the presence of the last χ-term. This latter term yields an unconstrained canonical momentum $p_\theta = i(\chi + \not{p}\theta)$ (since χ is arbitrary) and, therefore, the action (28) does not possess the fermionic κ-gauge symmetry.[e]

In order to eliminate the term $i\chi\dot{\theta}$ in (28) we need to gauge the subgroup $H(sp) \equiv \{\hat{\Theta}\}$ of $G(sp)$ (21) generated by the fermionic generator $\hat{\Theta}_\alpha$, in such a way that the resulting action would be independent of the corresponding group parameter χ^α. Thus we are lead to reducing the model on the coset space:

$$H(sp) \setminus O(sp) = \{\hat{\Theta}\} \setminus \left(\frac{\{\hat{T}\} \ltimes (SHW)}{\{\hat{T}\}} \right). \tag{29}$$

Now, taking into account Eq. (22) and

$$y(h(g)) = d\chi^\alpha \hat{\Theta}_\alpha , \quad h(g) = \exp \chi^\alpha \hat{\Theta}_\alpha \tag{30}$$

we easily find that the term

$$\int \langle S(g) | \, y(h(g)) \rangle = i \int dt \dot{\chi}^\alpha \theta_\alpha \tag{31}$$

in the coset action (15) precisely cancels (up to a total derivative) the unwanted χ-term in (28):

$$W_{H(sp)\setminus O(sp)}[g] = W_{BS} = \int dt \left[p^\mu (\dot{q}_\mu + i\theta\gamma_\mu \dot{\theta}) - \frac{1}{2}\dot{\omega}p^2 \right]. \tag{32}$$

4. Green-Schwarz Superstring

The construction in the previous section can easily be generalized to the case of GS superstring (we consider for definiteness the heterotic model). Now the relevant infinite-dimensional group is given by Eq. (1) $G(\mathrm{sstr}) = (\mathrm{Vir})^2 \ltimes (\mathcal{MSHW})$. Here $\hat{T}_{L,R}(x)$ are generators of the left- and right-handed Virasoro groups, while $\{\hat{P}^\mu(x), \hat{X}^\mu(x), \hat{Q}^\alpha(x), \hat{\Theta}_\alpha(x), \hat{S}^\mu(x)\}$ generate the Kac-Moody group $(\mathcal{MSHW})$. The

[e] In the Hamiltonian formalism the κ-gauge symmetry is generated by the Lorentz-*non*covariant first-class half of the fermionic constraint $d^\alpha \equiv -ip_\theta^\alpha - \not{p}^{\alpha\beta}\theta^\beta = 0$; ; see, e.g. Ref. 3.

last Kac-Moody generator $\hat{S}^\mu(x)$ is a bosonic Lorentz-vector[f] and its presence is crucial for closure of the algebra (i.e., fulfilment of the Jacobi identities).

The Lie-algebra of (1) explicitly reads (only non-zero commutators are displayed; primes indicate differentiation with respect to x)

$$
\begin{aligned}
[\hat{T}_{\mathrm{L,R}}(x), \hat{T}_{\mathrm{L,R}}(y)] &= \mp (2\hat{T}_{\mathrm{L,R}}(x)\, \delta'(x-y) + \hat{T}'_{\mathrm{L,R}}(x)\, \delta(x-y)) , \\
[\hat{T}_{\mathrm{L,R}}(x), (\hat{P} \mp \hat{X}')^\mu(y)] &= \mp(\hat{P} \mp \hat{X}')^\mu(x)\, \delta'(x-y), \\
[\hat{T}_{\mathrm{R}}(x), \hat{S}^\mu(y)] &= \hat{S}^\mu(x)\, \delta'(x-y) , \\
[\hat{T}_{\mathrm{R}}(x), \hat{Q}^\alpha(y)] &= \hat{Q}^\alpha(x)\, \delta'(x-y), \\
[\hat{T}_{\mathrm{R}}(x), \hat{\Theta}_\alpha(y)] &= -\hat{\Theta}'_\alpha(x)\, \delta(x-y) , \\
\{\hat{Q}^\alpha(x), \hat{Q}^\beta(x)\} &= 2i(\gamma^\mu)^{\alpha\beta}(\hat{P}_\mu + \hat{X}'_\mu + \hat{S}_\mu)(x)\, \delta(x-y) , \\
[\hat{Q}^\alpha(x), \hat{X}^\mu(y)] &= (\gamma^\mu)^{\alpha\beta}\hat{\Theta}_\beta(x)\, \delta(x-y), \\
[\hat{Q}^\alpha(x), \hat{P}^\mu(y)] &= -(\gamma^\mu)^{\alpha\beta}\hat{\Theta}_\beta(x)\, \delta'(x-y) , \\
[\hat{Q}^\alpha(x), \hat{S}^\mu(y)] &= 2(\gamma^\mu)^{\alpha\beta}\hat{\Theta}_\beta(x)\, \delta'(x-y) + 4(\gamma^\mu)^{\alpha\beta}\hat{\Theta}'_\beta(x)\, \delta(x-y) , \\
\{\hat{Q}^\alpha(x), \hat{\Theta}_\beta(y)\} &= -i\delta^\alpha_\beta \delta(x-y) , \\
[\hat{X}^\mu(x), \hat{P}^\nu(y)] &= \eta^{\mu\nu}\delta(x-y) .
\end{aligned}
\tag{33}
$$

The group element g and the Maurer-Cartan form $y(g^{-1})$ read formally

$$
\begin{aligned}
g = \exp\xi = \exp\int dx\, [&\omega_{\mathrm{L}}(x)\hat{T}_{\mathrm{L}}(x) + \omega_{\mathrm{R}}(x)\hat{T}_{\mathrm{R}}(x) + p_\mu(x)\hat{X}^\mu(x) + q_\mu(x)\hat{P}^\mu(x) \\
&+ \theta_\alpha(x)\hat{Q}^\alpha(x) + \chi^\alpha(x)\hat{\Theta}_\alpha(x) + \lambda_\mu(x)\hat{S}^\mu(x)] ,
\end{aligned}
\tag{34}
$$

$$
\begin{aligned}
-y(g^{-1}) = \int dx[&\tilde{\omega}_{\mathrm{L}}(x)\, \tilde{T}_{\mathrm{L}}(x)\; \tilde{\omega}_{\mathrm{R}}(x)\, \tilde{T}_{\mathrm{R}}(x) + \tilde{p}_\mu(x)\, \hat{X}^\mu(x) + \tilde{q}_\mu(x)\, \hat{P}^\mu(x) \\
&+ \tilde{\theta}_\alpha(x)\, \hat{Q}^\alpha(x) + \tilde{\chi}^\alpha(x)\, \tilde{\Theta}_\alpha(x) + \tilde{\lambda}_\mu(x)\, \tilde{S}^\mu(x)] ,
\end{aligned}
\tag{35}
$$

where the coefficients $\tilde{\omega}_{\mathrm{L,R}}(x),\ldots, \tilde{\lambda}_\mu(x)$ are one-form. The string analog of Eq. (23) becomes

$$
s(\xi) = \int dx[q_\mu(x)\hat{X}^{*\mu}(x) - p_\mu(x)\hat{P}^{*\mu}(x) + i\chi^\alpha(x)\hat{Q}_{*\alpha}(x) + i\theta_\alpha(x)\hat{\Theta}^{*\alpha}(x)] . \tag{36}
$$

From Eq. (36) we infer that the stationary subgroup of the co-orbit $\mathcal{O}$(sstr) (10) of the group G(sstr) (1) is

$$
G_{\mathrm{stat}}(\mathrm{sstr}) = (\mathrm{Vir})^2 \ltimes \{\hat{S}^\mu(\cdot)\} , \quad \text{i.e.,} \quad \mathcal{O}(\mathrm{sstr}) = \frac{(\mathrm{Vir})^2 \ltimes (\mathcal{MSHW})}{(\mathrm{Vir})^2 \ltimes \{\hat{S}^\mu(\cdot)\}} . \tag{37}
$$

Here $\{\hat{S}^\mu(.)\}$ is the Kac-Moody subgroup generated by $\hat{S}^\mu(x)$.

[f] In the superparticle (zero mode) limit, $\hat{P}^\mu = \int dx(\hat{P}^\mu(x) + \hat{S}^\mu(x))$ and $\hat{S}^\mu = \int dx \hat{S}^\mu(x)$ decouples from (trivially commutes with) the superparticle algebra (20).

Accounting for Eqs. (35) and (36), for the corresponding co-orbit action (13) we have (cf. (26))

$$W_{O(\mathrm{sstr})}[g] = \int d^{-1}\Big(\int dx[\tilde{p}^{\mu}(x)\wedge\tilde{q}_{\mu}(x) + i\tilde{\theta}_{\alpha}(x)\wedge\tilde{\chi}^{\alpha}(x)]\Big). \tag{38}$$

As in the superparticle case the solution for $y(g^{-1})$ (35) in terms of the group parameters of g (34) can be easily found by means of the Maurer-Cartan equation, (7). The solution reads

$$\tilde{\omega}_{\mathrm{L,R}} = -\frac{df_{\mathrm{L,R}}}{\partial_x f_{\mathrm{L,R}}}, \quad f_{\mathrm{L,R}}(x) = [\exp(\mp\omega_{\mathrm{L,R}}(x)\partial_x)]\,x\,, \tag{39}$$

$$\tilde{\theta} = d\theta + \tilde{\omega}_{\mathrm{R}}\partial_x\theta\,, \quad \tilde{\lambda}^{\mu} = d\lambda^{\mu} + \tilde{\omega}_{\mathrm{R}}\partial_x\lambda^{\mu} + i\theta\gamma^{\mu}\tilde{\theta}\,, \tag{40}$$

$$\tilde{p}^{\mu} = dp^{\mu} + \partial_{\mu}\left[\frac{1}{2}\,\tilde{\omega}_{\mathrm{R}}(p^{\mu} - \partial_x q^{\mu}) - \frac{1}{2}\,\tilde{\omega}_{\mathrm{L}}(p^{\mu} + \partial_x q^{\mu}) - i\theta\gamma^{\mu}(d\theta + \tilde{\omega}_{\mathrm{R}}\partial_x\theta)\right], \tag{41}$$

$$\tilde{q}^{\mu} = dq^{\mu} - \frac{1}{2}\tilde{\omega}_{\mathrm{R}}(p^{\mu} - \partial_x q^{\mu}) - \frac{1}{2}\tilde{\omega}_{\mathrm{L}}(p^{\mu} + \partial_x q^{\mu}) + i\theta\gamma^{\mu}(d\theta + \tilde{\omega}_{\mathrm{R}}\partial_x\theta), \tag{42}$$

$$\tilde{\chi} = d\chi + \partial_x(\tilde{\omega}_{\mathrm{R}}\chi) + (p^{\mu} - \partial_x q^{\mu} - 2\lambda^{\mu})\gamma_{\mu}\tilde{\theta} - 4\lambda_{\mu}\gamma^{\mu}\partial_x\tilde{\theta} + 4i(\partial_x\theta\gamma^{\mu}\tilde{\theta})\gamma_{\mu}\theta\,. \tag{43}$$

In (39) $f_{\mathrm{L,R}}(x)$ denote the (inverse) diffeomorphisms generated by the Virasoro generators $\hat{T}_{\mathrm{L,R}}(x)$ (cf. Refs. 7 and 10).

Substituting the solutions (39)–(43) into (38) we get the following co-orbit action,

$$W_{O(\mathrm{sstr})}[g] = \int\Bigg(\int dx[\,p^{\mu}dq_{\mu} - \tilde{\omega}_{\mathrm{L}}\,\frac{1}{4}(p + \partial_x q)^2 - \tilde{\omega}_{\mathrm{R}}\,\frac{1}{4}(p - \partial_x q - 2i\theta\gamma\partial_x\theta)^2 + i(p^{\mu} - \partial_x q^{\mu} - i\theta\gamma^{\mu}\partial_x\theta)\,(\theta\gamma_{\mu}d\theta) - \chi(d\theta + \tilde{\omega}_{\mathrm{R}}\partial_x\theta)]\Bigg). \tag{44}$$

The co-orbit action (44) is manifestly invariant under the gauge group $G_{\mathrm{stat}}(sp)$ (37): it is reparametrization invariant and does not explicitly depend on the group coordinate $\lambda^{\mu}(x)$.

The action (44) differs from the (Hamiltonian first-order form of the) GS superstring action due to the presence of the last χ-term. As in the superparticle case, this χ-term breaks the fermionic κ-gauge invariance. Similarly to the superparticle case, however, we can get rid of the χ-term in (44) by restricting the action on the coset space:

$$H(\mathrm{sstr})\setminus\left(\frac{G(\mathrm{sstr})}{G_{\mathrm{stat}}(\mathrm{sstr})}\right) = \{\hat{\Theta}(\cdot)\}\setminus\left(\frac{(\mathrm{Vir})^2\ltimes(\mathcal{MSHW})}{(\mathrm{Vir})^2\ltimes\{\hat{S}^{\mu}(\cdot)\}}\right). \tag{45}$$

Similarly to the superparticle case, $H(\mathrm{sstr}) \equiv \{\hat{\Theta}(\cdot)\}$ in (45) denotes the abelian Kac-Moody subgroup generated by the fermionic generator $\hat{\Theta}_{\alpha}(x)$, i.e.,

$$h(g) = \exp\int dx\,\chi^{\alpha}(x)\hat{\Theta}_{\alpha}(x)\,, \quad y(h(g)) = \int dx\,d\chi^{\alpha}(x)\hat{\Theta}_{\alpha}(x)\,. \tag{46}$$

In order to calculate the additional local term in the coset geometric action (15) we need only the coefficient in front of $\hat{\Theta}^*(x)$ in the expansion of the integrated "anomaly" $S(g)$ with respect to the dual basis, which (in complete analogy with the superparticle Eq. (22)) is easily found to be:

$$S(g)=\int dx[i\theta_{\alpha}^{(\omega_R)}(x)\,\hat{\Theta}^{*\alpha}(x)+\ldots], \tag{47}$$

$$\theta^{(\omega_R)}(x)=\left[\sum_{n=0}^{\infty}\frac{1}{(n+1)!}(\omega_R(x)\,\partial_x)^n\right]\theta(x). \tag{48}$$

The last term in the coset action (15) becomes

$$\int\langle S(g)|\,y(h(g))\rangle=i\int\left(\int dx[d\chi(x)\,\theta^{(\omega_R)}(x)]\right)=i\int\left(\int dx[d\chi+\partial_x(\tilde{\omega}_R\chi)]\,\theta\right), \tag{49}$$

where we used Eqs. (39) to get the last equality. Therefore, as in the superparticle case, the term (49) precisely cancels (up to total derivatives) the unwanted χ-term in (44):

$$W_{H(\mathrm{sstr})\backslash O(\mathrm{sstr})}[g]=W_{\text{Green-Schwarz}}=\int\Bigg(\int dx\Bigg[p^{\mu}dq_{\mu}+\frac{df_{\mathrm{L}}}{\partial_x f_{\mathrm{L}}}\frac{1}{4}(p+\partial_x q)^2 +\frac{df_{\mathrm{R}}}{\partial_x f_{\mathrm{R}}}\frac{1}{4}(p-\partial_x q-2i\theta\gamma\partial_x\theta)^2 + i(p^{\mu}-\partial_x q^{\mu}-i\theta\gamma^{\mu}\partial_x\theta)(\theta\gamma_{\mu}d\theta)\Bigg]\Bigg). \tag{50}$$

Obviously, the action (50) is related to the standard (Hamiltonian form of the) GS action (see, e.g. Ref. 3) by the change of variables $\partial_t f_{\mathrm{L,R}}/\partial_x f_{\mathrm{L,R}}=-\Lambda_{\mathrm{L,R}}$, where $\Lambda_{\mathrm{L,R}}$ are the Lagrange multipliers of the reparametrization constraints.

After having reformulated the GS superstring as a geometric action on infinite-dimensional group coset space, it now becomes possible to use the techniques of the coadjoint orbit method to calculate correlation functions within the functional integral representation. A particularly important role will be played by the fundamental group composition laws (14). These questions are now being studied.

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